Title: An Elegant Method to Prove Fermat's Last Theorem
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Abstract and Introduction:

In number theory, Fermat's Last Theorem states that no three positive integers, X, Y and Z satisfy the equation $X^n + Y^n = Z^n$ for any integer value of n greater than 2. The cases n = 1 and n = 2 have been known since antiquity to have infinite number of solutions [1].

The proposition was first conjectured by Pierre de Fermat in 1637 in the margin of a copy of Arithmetica; Fermat added that he had a proof that was too large to fit in the margin. However, there were first doubts about it since the publication was done by his son without his consent, after Fermat' death [2]. After 358 years of effort by mathematicians, the first successful proof was released in 1994 by Andrew Wiles, and formally published in 1995; it was described as a "stunning advance" in the citation for Wiles's Abel Prize award in 2016 [3].

He was successful in proving it by using the elliptic curve and Taniyama-Shimura conjecture on more than 250 pages, which method is rather complicated. However, I have found an elegant method to solve the theorem, which I think can be understood by those mathematicians who are not always specialists in number theory. I present here to prove it simply by using a combination method and an even-odd number method.

Key words: Number theory, Fermat' last theorem, combination, even number and odd number

Fermat's Last Theorem is that when integers X, Y and Z are more than 0 with an integer n more than 2, there are no appropriate integers X, Y and Z which meet the formula bellow

$$X^n + Y^n = Z^n \quad \cdots ①\text{-}1$$

[no common divisor (>1) among X, Y and Z ・・・①-2]

When X = Y, $2X^n = Z^n$. Thus, Z must be an even number. Z can be expressed as $2Z_1$.

$$\therefore 2X^n = (2Z_1)^n = 2^n(Z_1)^n$$

$$\therefore X^n = 2^{n-1}(Z_1)^n$$

From this, X must be an even number. This means that all of X, Y and Z are even numbers, which contradicts the determined condition ①-2.

$$\therefore X \neq Y$$

(1) When Z is an even number (expressed as $2Z_1$).

Both X and Y must be either odd numbers, or even numbers. However, there is no possibility that they are both even numbers. Because, if they are both even numbers, X, Y and Z are all even numbers, which contradicts ①-2. Therefore, study needs for X and Y which are both odd numbers. X, Y and Z can be expressed as bellow.

$X = m - d$, $Y = m + d$, $Z = 2Z_1$ [Z_1: integer >0]

[m, d: one is an even number, the other is an odd number. m>d>0]

The formula ①-1 can be expressed as bellow.

$$(m - d)^n + (m + d)^n = (2Z_1)^n \quad \cdots ②\text{-}1$$

$$\therefore (2Z_1)^n = \{m^n + {}_nC_1\, m^{n-1}(-d) + {}_nC_2\, m^{n-2}(-d)^2 + {}_nC_3\, m^{n-3}(-d)^3$$
$$+ {}_nC_4\, m^{n-4}(-d)^4 + \cdots$$
$$\cdots + {}_nC_{n-4}\, m^4(-d)^{n-4} + {}_nC_{n-3}\, m^3(-d)^{n-3} + {}_nC_{n-2}\, m^2(-d)^{n-2}$$

$+ {}_nC_{n-1}\, m(-d)^{n-1} + (-d)^n \}$

$+ \{m^n + {}_nC_1\, m^{n-1}\, d + {}_nC_2\, m^{n-2}\, d^2 + {}_nC_3\, m^{n-3}\, d^3 + {}_nC_4\, m^{n-4}\, d^4 + \cdot\cdot\cdot$

$\cdot\cdot\cdot + {}_nC_{n-4}\, m^4\, d^{n-4} + {}_nC_{n-3}\, m^3\, d^{n-3} + {}_nC_{n-2}\, m^2\, d^{n-2}$

$+ {}_nC_{n-1}\, m\, d^{n-1} + d^n \}$ ・・・③-1

(1)-A: When n is an even number,

$(2Z_1)^n = \{m^n - {}_nC_1\, m^{n-1}\, d + {}_nC_2\, m^{n-2}\, d^2 - {}_nC_3\, m^{n-3}\, d^3 + {}_nC_4\, m^{n-4}\, d^4 - \cdot\cdot\cdot$

$\cdot\cdot\cdot + {}_nC_{n-4}\, m^4\, d^{n-4} - {}_nC_{n-3}\, m^3\, d^{n-3} + {}_nC_{n-2}\, m^2\, d^{n-2} - {}_nC_{n-1}\, md^{n-1} + d^n\}$

$+\{m^n + {}_nC_1\, m^{n-1}\, d + {}_nC_2\, m^{n-2}\, d^2 + {}_nC_3\, m^{n-3}\, d^3 + {}_nC_4\, m^{n-4}\, d^4 + \cdot\cdot\cdot$

$\cdot\cdot\cdot + {}_nC_{n-4}\, m^4\, d^{n-4} + {}_nC_{n-3}\, m^3\, d^{n-3} + {}_nC_{n-2}\, m^2\, d^{n-2} + {}_nC_{n-1}\, m\, d^{n-1} + d^n\}$

$= 2\{m^n + {}_nC_2\, m^{n-2}\, d^2 + {}_nC_4\, m^{n-4}\, d^4 + \cdot\cdot\cdot$

$\cdot\cdot\cdot + {}_nC_{n-4}\, m^4\, d^{n-4} + {}_nC_{n-2}\, m^2\, d^{n-2} + d^n\}$

When divided by 2,

$2^{n-1}(Z_1)^n = m^n + {}_nC_2\, m^{n-2}\, d^2 + {}_nC_4\, m^{n-4}\, d^4 + \cdot\cdot\cdot$

$\cdot\cdot\cdot + {}_nC_{n-4}\, m^4\, d^{n-4} + {}_nC_{n-2}\, m^2\, d^{n-2} + d^n$ ・・・③-2

(1)-A-1: When m is an even number and d is an odd number, all the other terms are even numbers except for d^n, which is an odd number. Therefore, in the formula ③-2, the left side value ≠ the right side value.

$\therefore (m - d)^n + (m + d)^n \neq (2Z_1)^n$ ・・・②-2

(1)-A-2: When m is an odd number and d is an even number, all the other terms are even number except for m^n, which is an odd number. Therefore, in the formula ③-2, the left side value ≠ the right side value.

$\therefore (m - d)^n + (m + d)^n \neq (2Z_1)^n$ ・・・②-2

(1)-B: When n is an odd number, from ③-1

$\therefore (2Z_1)^n == \{m^n - {}_nC_1\, m^{n-1}d + {}_nC_2\, m^{n-2}\, d^2 - {}_nC_3\, m^{n-3}\, d^3$

$+ {}_nC_4\, m^{n-4}\, d^4 + \cdot\cdot\cdot$

$\cdot\cdot\cdot - {}_nC_{n-4}\, m^4\, d^{n-4} + {}_nC_{n-3}\, m^3\, d^{n-3} - {}_nC_{n-2}\, m^2\, d^{n-2}$

$+ {}_nC_{n-1}\, md^{n-1} - d^n\}$

$+ \{m^n + {}_nC_1\, m^{n-1}\, d + {}_nC_2\, m^{n-2}\, d^2 + {}_nC_3\, m^{n-3}\, d^3 + {}_nC_4\, m^{n-4}\, d^4 + \cdot\cdot\cdot$

$\cdot\cdot\cdot + {}_nC_{n-4}\, m^4\, d^{n-4} + {}_nC_{n-3}\, m^3\, d^{n-3} + {}_nC_{n-2}\, m^2\, d^{n-2} + {}_nC_{n-1}\, m\, d^{n-1} + d^n\}$

$= 2\{m^n + {}_nC_2\, m^{n-2}\, d^2 + {}_nC_4\, m^{n-4}\, d^4 + \cdot\cdot\cdot$

$\cdot\cdot\cdot + {}_nC_{n-5}\, m^5\, d^{n-5} + {}_nC_{n-3}\, m^3\, d^{n-3} + {}_nC_{n-1}\, md^{n-1}\}$

When divided by 2,

$\therefore 2^{n-1}(Z_1)^n = m^n + {}_nC_2\, m^{n-2}\, d^2 + {}_nC_4\, m^{n-4}\, d^4 + \cdot\cdot\cdot$

$\cdot\cdot\cdot + {}_nC_{n-3}\, m^3\, d^{n-3} + {}_nC_{n-1}\, md^{n-1}$ $\cdot\cdot\cdot$ ③-3

(1)-B-1: When m is an odd number, and d is an even number, all the other terms except for m^n are even numbers. Therefore, in the formula ③-3, the left side value $\neq$ the right side value.

$\therefore (m - d)^n + (m + d)^n \neq (2Z_1)^n$ $\cdot\cdot\cdot$ ②-2

(1)-B-2: When m is an even number, and d is an odd number. The m and d can be expressed as bellow.

$m = 2^j k$ [j; integer>0, k; odd number>0]

$d = 2d_1 - 1$ [d_1: integer>0]

$\therefore d^{n-1} = (2d_1 - 1)^{n-1}$

$= (2d_1)^{n-1} - {}_nC_1(2d_1)^{n-2} + {}_nC_2\, (2d_1)^{n-3} - {}_nC_3\, (2d_1)^{n-4} + \cdot\cdot\cdot$

$- {}_nC_{n-4}(2d_1)^3 + {}_nC_{n-3}\, (2d_1)^2 - {}_nC_{n-2}\, (2d_1) + 1$

The $Z = 2Z_1$ can be also expressed as bellow.

$Z = 2^i\, (Z_2)$ [i: integer>0, Z_2; odd number>0]

From ③-3,

$\therefore \{2^i\, (Z_2)\, \}^n /2 = 2^{in-1}\, (Z_2)^n$

$= m^n + {}_nC_2\, m^{n-2}\, d^2 + \cdot\cdot\cdot + {}_nC_{n-3}\, m^3\, d^{n-3}$

$+nm\{(2d_1)^{n-1} - {}_nC_1(2d_1)^{n-2} + {}_nC_2\, (2d_1)^{n-3} - {}_nC_3\, (2d_1)^{n-4} + \cdot\cdot\cdot$

$- {}_nC_{n-4}(2d_1)^3 + {}_nC_{n-3}\, (2d_1)^2 - {}_nC_{n-2}\, (2d_1) + 1\}$

$= (2^j k)^n + {}_nC_2\, (2^j k)^{n-2}\, d^2 + \cdot\cdot\cdot + {}_nC_{n-3}\, (2^j k)^3\, d^{n-3}$

$+ (2^j k)n\{(2d_1)^{n-1} - {}_nC_1(2d_1)^{n-2} + {}_nC_2\, (2d_1)^{n-3} - {}_nC_3\, (2d_1)^{n-4} + \cdot\cdot\cdot$

$- {}_nC_{n-4}(2d_1)^3 + {}_nC_{n-3}\, (2d_1)^2 - {}_nC_{n-2}\, (2d_1) + 1\}$

$= 2^{jn}\, k^n + {}_nC_2\, 2^{j(n-2)}\, k^{n-2}\, d^2 + \cdot\cdot\cdot + {}_nC_{n-3}\, 2^{3j}\, k^3\, d^{n-3}$

$+ 2^j\, kn\{(2d_1)^{n-1} - {}_nC_1(2d_1)^{n-2} + {}_nC_2\, (2d_1)^{n-3} - {}_nC_3\, (2d_1)^{n-4} + \cdot\cdot\cdot$

$- {}_nC_{n-4}(2d_1)^3 + {}_nC_{n-3}\, (2d_1)^2 - {}_nC_{n-2}\, (2d_1) + 1\}$

When $(in - 1) - j > 0$, divided by 2^j

$\therefore 2^{in-1-j}\, (Z_2)^n = 2^{j(n-1)}\, k^n + {}_nC_2\, 2^{j(n-3)}\, k^{n-2}\, d^2 + \cdot\cdot\cdot + {}_nC_{n-3}\, 2^{2j}\, k^3\, d^{n-3}$

$+ kn\{(2d_1)^{n-1} - {}_nC_1(2d_1)^{n-2} + {}_nC_2\, (2d_1)^{n-3} - {}_nC_3\, (2d_1)^{n-4} + \cdot\cdot\cdot$

$- {}_nC_{n-4}(2d_1)^3 + {}_nC_{n-3}\, (2d_1)^2 - {}_nC_{n-2}\, (2d_1)\} + kn$

Since both k and n are odd numbers, kn is an odd number. All the other terms except for kn are even numbers. Therefore, the left side value $\neq$

the right side value.

$$\therefore (m - d)^n + (m + d)^n \neq (2Z_1)^n \quad \cdots ②\text{-}2$$

Consequently, from 1-A) and 1-B),

$$\therefore X^n + Y^n \neq Z^n \quad \cdots ①\text{-}0$$

(2) When Z is an odd number:

From ②-2,

$$(m - d)^n + (m + d)^n \neq (2Z_1)^n$$

$$\therefore (m - d)^n \neq (2Z_1)^n - (m + d)^n = -\{(m + d)^n - (2Z_1)^n\}$$

$$\therefore (m - d)^{2n} \neq \{(m + d)^n - (2Z_1)^n\}^2$$

$$\therefore (m - d)^n \neq (m + d)^n - (2Z_1)^n$$

$$\therefore (2Z_1)^n + (m - d)^n \neq (m + d)^n$$

These $2Z_1$, $(m - d)$ and $(m + d)$ can be replaced by $X = 2Z_1$, $Y = m - d$ and $Z = m + d$, respectively, because these conditions for X, Y and Z are exactly what is required for the case (2).

$$\therefore X^n + Y^n \neq Z^n \quad \cdots ①\text{-}0$$

From (1) and (2), the formula ①-1 is incorrect, meaning that Fermat's last theorem is correct.

References:

[1]The 1670 edition of diophantus's Arithmetica includes Fermat's commentary, referred to as his "Last Theorem, post published by his son.

[2]Nigel Boston p.5 "The proof of Fermat's last theorem" (https://www.math.wisc.edu/～boston/869. pdf)

[3]Abel prize 2016-full citation (http://www.abelprize.no/c67107/binfil/download. Php tid=67059)